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Diploma Programme

Mathematics SL guide

First examinations 2014

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The International Baccalaureate Organization (known as the IB) offers three high-quality and challenging educational programmes for a worldwide community of schools, aiming to create a better, more peaceful world. This publication is one of a range of materials produced to support these programmes.

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IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.

IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

IB learners strive to be:

Inquirers	They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.
Knowledgeable	They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.
Thinkers	They exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.
Communicators	They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.
Principled	They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.
Open-minded	They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience.
Caring	They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.
Risk-takers	They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.
Balanced	They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.
Reflective	They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

Contents

Purpose of this document

This publication is intended to guide the planning, teaching and assessment of the subject in schools. Subject teachers are the primary audience, although it is expected that teachers will use the guide to inform students and parents about the subject.

This guide can be found on the subject page of the online curriculum centre (OCC) at http://occ.ibo.org, a password-protected IB website designed to support IB teachers. It can also be purchased from the IB store at http://store.ibo.org.

Additional resources

Additional publications such as teacher support materials, subject reports, internal assessment guidance and grade descriptors can also be found on the OCC. Specimen and past examination papers as well as markschemes can be purchased from the IB store.

Teachers are encouraged to check the OCC for additional resources created or used by other teachers. Teachers can provide details of useful resources, for example: websites, books, videos, journals or teaching ideas.

First examinations 2014

The Diploma Programme

The Diploma Programme is a rigorous pre-university course of study designed for students in the 16 to 19 age range. It is a broad-based two-year course that aims to encourage students to be knowledgeable and inquiring, but also caring and compassionate. There is a strong emphasis on encouraging students to develop intercultural understanding, open-mindedness, and the attitudes necessary for them to respect and evaluate a range of points of view.

The Diploma Programme hexagon

The course is presented as six academic areas enclosing a central core (see figure 1). It encourages the concurrent study of a broad range of academic areas. Students study: two modern languages (or a modern language and a classical language); a humanities or social science subject; an experimental science; mathematics; one of the creative arts. It is this comprehensive range of subjects that makes the Diploma Programme a demanding course of study designed to prepare students effectively for university entrance. In each of the academic areas students have flexibility in making their choices, which means they can choose subjects that particularly interest them and that they may wish to study further at university.

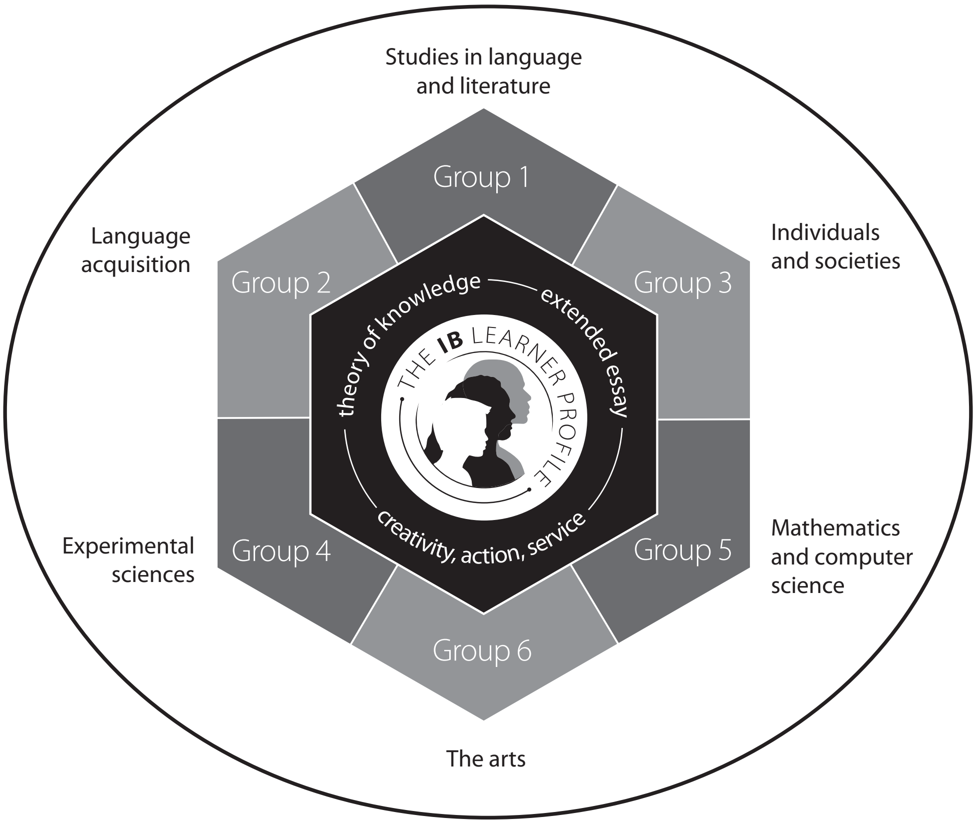

Figure 1

Diploma Programme model

Choosing the right combination

Students are required to choose one subject from each of the six academic areas, although they can choose a second subject from groups 1 to 5 instead of a group 6 subject. Normally, three subjects (and not more than four) are taken at higher level (HL), and the others are taken at standard level (SL). The IB recommends 240 teaching hours for HL subjects and 150 hours for SL. Subjects at HL are studied in greater depth and breadth than at SL.

At both levels, many skills are developed, especially those of critical thinking and analysis. At the end of the course, students' abilities are measured by means of external assessment. Many subjects contain some element of coursework assessed by teachers. The courses are available for examinations in English, French and Spanish, with the exception of groups 1 and 2 courses where examinations are in the language of study.

The core of the hexagon

All Diploma Programme students participate in the three course requirements that make up the core of the hexagon. Reflection on all these activities is a principle that lies at the heart of the thinking behind the Diploma Programme.

The theory of knowledge course encourages students to think about the nature of knowledge, to reflect on the process of learning in all the subjects they study as part of their Diploma Programme course, and to make connections across the academic areas. The extended essay, a substantial piece of writing of up to 4,000 words, enables students to investigate a topic of special interest that they have chosen themselves. It also encourages them to develop the skills of independent research that will be expected at university. Creativity, action, service involves students in experiential learning through a range of artistic, sporting, physical and service activities.

The IB mission statement and the IB learner profile

The Diploma Programme aims to develop in students the knowledge, skills and attitudes they will need to fulfill the aims of the IB, as expressed in the organization's mission statement and the learner profile. Teaching and learning in the Diploma Programme represent the reality in daily practice of the organization's educational philosophy.

Nature of the subject

Introduction

The nature of mathematics can be summarized in a number of ways: for example, it can be seen as a well-defined body of knowledge, as an abstract system of ideas, or as a useful tool. For many people it is probably a combination of these, but there is no doubt that mathematical knowledge provides an important key to understanding the world in which we live. Mathematics can enter our lives in a number of ways: we buy produce in the market, consult a timetable, read a newspaper, time a process or estimate a length. Mathematics, for most of us, also extends into our chosen profession: visual artists need to learn about perspective; musicians need to appreciate the mathematical relationships within and between different rhythms; economists need to recognize trends in financial dealings; and engineers need to take account of stress patterns in physical materials. Scientists view mathematics as a language that is central to our understanding of events that occur in the natural world. Some people enjoy the challenges offered by the logical methods of mathematics and the adventure in reason that mathematical proof has to offer. Others appreciate mathematics as an aesthetic experience or even as a cornerstone of philosophy. This prevalence of mathematics in our lives, with all its interdisciplinary connections, provides a clear and sufficient rationale for making the study of this subject compulsory for students studying the full diploma.

Summary of courses available

Because individual students have different needs, interests and abilities, there are four different courses in mathematics. These courses are designed for different types of students: those who wish to study mathematics in depth, either as a subject in its own right or to pursue their interests in areas related to mathematics; those who wish to gain a degree of understanding and competence to understand better their approach to other subjects; and those who may not as yet be aware how mathematics may be relevant to their studies and in their daily lives. Each course is designed to meet the needs of a particular group of students. Therefore, great care should be taken to select the course that is most appropriate for an individual student.

In making this selection, individual students should be advised to take account of the following factors:

- their own abilities in mathematics and the type of mathematics in which they can be successful
- their own interest in mathematics and those particular areas of the subject that may hold the most interest for them
- their other choices of subjects within the framework of the Diploma Programme
- their academic plans, in particular the subjects they wish to study in future
- their choice of career.

Teachers are expected to assist with the selection process and to offer advice to students.

Mathematical studies SL

This course is available only at standard level, and is equivalent in status to mathematics SL, but addresses different needs. It has an emphasis on applications of mathematics, and the largest section is on statistical techniques. It is designed for students with varied mathematical backgrounds and abilities. It offers students opportunities to learn important concepts and techniques and to gain an understanding of a wide variety of mathematical topics. It prepares students to be able to solve problems in a variety of settings, to develop more sophisticated mathematical reasoning and to enhance their critical thinking. The individual project is an extended piece of work based on personal research involving the collection, analysis and evaluation of data. Students taking this course are well prepared for a career in social sciences, humanities, languages or arts. These students may need to utilize the statistics and logical reasoning that they have learned as part of the mathematical studies SL course in their future studies.

Mathematics SL

This course caters for students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.

Mathematics HL

This course caters for students with a good background in mathematics who are competent in a range of analytical and technical skills. The majority of these students will be expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. Others may take this subject because they have a strong interest in mathematics and enjoy meeting its challenges and engaging with its problems.

Further mathematics HL

This course is available only at higher level. It caters for students with a very strong background in mathematics who have attained a high degree of competence in a range of analytical and technical skills, and who display considerable interest in the subject. Most of these students will expect to study mathematics at university, either as a subject in its own right or as a major component of a related subject. The course is designed specifically to allow students to learn about a variety of branches of mathematics in depth and also to appreciate practical applications. It is expected that students taking this course will also be taking mathematics HL.

Note: Mathematics HL is an ideal course for students expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering or technology. It should not be regarded as necessary for such students to study further mathematics HL. Rather, further mathematics HL is an optional course for students with a particular aptitude and interest in mathematics, enabling them to study some wider and deeper aspects of mathematics, but is by no means a necessary qualification to study for a degree in mathematics.

Mathematics SL—course details

The course focuses on introducing important mathematical concepts through the development of mathematical techniques. The intention is to introduce students to these concepts in a comprehensible and coherent way, rather than insisting on the mathematical rigour required for mathematics HL. Students should, wherever possible, apply the mathematical knowledge they have acquired to solve realistic problems set in an appropriate context.

The internally assessed component, the exploration, offers students the opportunity for developing independence in their mathematical learning. Students are encouraged to take a considered approach to various mathematical activities and to explore different mathematical ideas. The exploration also allows students to work without the time constraints of a written examination and to develop the skills they need for communicating mathematical ideas.

This course does not have the depth found in the mathematics HL courses. Students wishing to study subjects with a high degree of mathematical content should therefore opt for a mathematics HL course rather than a mathematics SL course.

Prior learning

Mathematics is a linear subject, and it is expected that most students embarking on a Diploma Programme (DP) mathematics course will have studied mathematics for at least 10 years. There will be a great variety of topics studied, and differing approaches to teaching and learning. Thus students will have a wide variety of skills and knowledge when they start the mathematics SL course. Most will have some background in arithmetic, algebra, geometry, trigonometry, probability and statistics. Some will be familiar with an inquiry approach, and may have had an opportunity to complete an extended piece of work in mathematics.

At the beginning of the syllabus section there is a list of topics that are considered to be prior learning for the mathematics SL course. It is recognized that this may contain topics that are unfamiliar to some students, but it is anticipated that there may be other topics in the syllabus itself that these students have already encountered. Teachers should plan their teaching to incorporate topics mentioned that are unfamiliar to their students.

Links to the Middle Years Programme

The prior learning topics for the DP courses have been written in conjunction with the Middle Years Programme (MYP) mathematics guide. The approaches to teaching and learning for DP mathematics build on the approaches used in the MYP. These include investigations, exploration and a variety of different assessment tools.

A continuum document called *Mathematics: The MYP–DP continuum* (November 2010) is available on the DP mathematics home pages of the OCC. This extensive publication focuses on the alignment of mathematics across the MYP and the DP. It was developed in response to feedback provided by IB World Schools, which expressed the need to articulate the transition of mathematics from the MYP to the DP. The publication also highlights the similarities and differences between MYP and DP mathematics, and is a valuable resource for teachers.

Mathematics and theory of knowledge

The *Theory of knowledge guide* (March 2006) identifies four ways of knowing, and it could be claimed that these all have some role in the acquisition of mathematical knowledge. While perhaps initially inspired by data from sense perception, mathematics is dominated by reason, and some mathematicians argue that their subject is a language, that it is, in some sense, universal. However, there is also no doubt that mathematicians perceive beauty in mathematics, and that emotion can be a strong driver in the search for mathematical knowledge.

As an area of knowledge, mathematics seems to supply a certainty perhaps missing in other disciplines. This may be related to the "purity" of the subject that makes it sometimes seem divorced from reality. However, mathematics has also provided important knowledge about the world, and the use of mathematics in science and technology has been one of the driving forces for scientific advances.

Despite all its undoubted power for understanding and change, mathematics is in the end a puzzling phenomenon. A fundamental question for all knowers is whether mathematical knowledge really exists independently of our thinking about it. Is it there "waiting to be discovered" or is it a human creation?

Students' attention should be drawn to questions relating theory of knowledge (TOK) and mathematics, and they should be encouraged to raise such questions themselves, in mathematics and TOK classes. This includes questioning all the claims made above. Examples of issues relating to TOK are given in the "Links" column of the syllabus. Teachers could also discuss questions such as those raised in the "Areas of knowledge" section of the TOK guide.

Mathematics and the international dimension

Mathematics is in a sense an international language, and, apart from slightly differing notation, mathematicians from around the world can communicate within their field. Mathematics transcends politics, religion and nationality, yet throughout history great civilizations owe their success in part to their mathematicians being able to create and maintain complex social and architectural structures.

Despite recent advances in the development of information and communication technologies, the global exchange of mathematical information and ideas is not a new phenomenon and has been essential to the progress of mathematics. Indeed, many of the foundations of modern mathematics were laid many centuries ago by Arabic, Greek, Indian and Chinese civilizations, among others. Teachers could use timeline websites to show the contributions of different civilizations to mathematics, but not just for their mathematical content. Illustrating the characters and personalities of the mathematicians concerned and the historical context in which they worked brings home the human and cultural dimension of mathematics.

The importance of science and technology in the everyday world is clear, but the vital role of mathematics is not so well recognized. It is the language of science, and underpins most developments in science and technology. A good example of this is the digital revolution, which is transforming the world, as it is all based on the binary number system in mathematics.

Many international bodies now exist to promote mathematics. Students are encouraged to access the extensive websites of international mathematical organizations to enhance their appreciation of the international dimension and to engage in the global issues surrounding the subject.

Examples of global issues relating to international-mindedness (**Int**) are given in the "Links" column of the syllabus.

Aims

Group 5 aims

The aims of all mathematics courses in group 5 are to enable students to:

1. enjoy mathematics, and develop an appreciation of the elegance and power of mathematics
2. develop an understanding of the principles and nature of mathematics
3. communicate clearly and confidently in a variety of contexts
4. develop logical, critical and creative thinking, and patience and persistence in problem-solving
5. employ and refine their powers of abstraction and generalization
6. apply and transfer skills to alternative situations, to other areas of knowledge and to future developments
7. appreciate how developments in technology and mathematics have influenced each other
8. appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics
9. appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular "area of knowledge" in the TOK course.

Assessment objectives

Problem-solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematics SL course, students will be expected to demonstrate the following.

1. **Knowledge and understanding**: recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
2. **Problem-solving**: recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems.
3. **Communication and interpretation**: transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation.
4. **Technology**: use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems.
5. **Reasoning**: construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.
6. **Inquiry approaches**: investigate unfamiliar situations, both abstract and real-world, involving organizing and analysing information, making conjectures, drawing conclusions and testing their validity.

Syllabus outline

Syllabus component	Teaching hours
	SL
All topics are compulsory. Students must study all the sub-topics in each of the topics in the syllabus as listed in this guide. Students are also required to be familiar with the topics listed as prior learning.	
Topic 1 Algebra	9
Topic 2 Functions and equations	24
Topic 3 Circular functions and trigonometry	16
Topic 4 Vectors	16
Topic 5 Statistics and probability	35
Topic 6 Calculus	40
Mathematical exploration Internal assessment in mathematics SL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics.	10
Total teaching hours	**150**

Approaches to the teaching and learning of mathematics SL

Throughout the DP mathematics SL course, students should be encouraged to develop their understanding of the methodology and practice of the discipline of mathematics. The processes of **mathematical inquiry**, **mathematical modelling and applications** and the **use of technology** should be introduced appropriately. These processes should be used throughout the course, and not treated in isolation.

Mathematical inquiry

The IB learner profile encourages learning by experimentation, questioning and discovery. In the IB classroom, students should generally learn mathematics by being active participants in learning activities rather than recipients of instruction. Teachers should therefore provide students with opportunities to learn through mathematical inquiry. This approach is illustrated in figure 2.

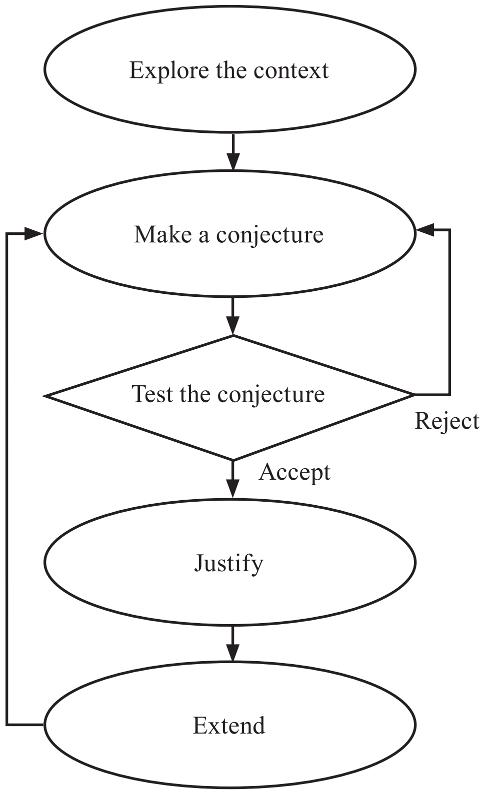

Figure 2

Mathematical modelling and applications

Students should be able to use mathematics to solve problems in the real world. Engaging students in the mathematical modelling process provides such opportunities. Students should develop, apply and critically analyse models. This approach is illustrated in figure 3.

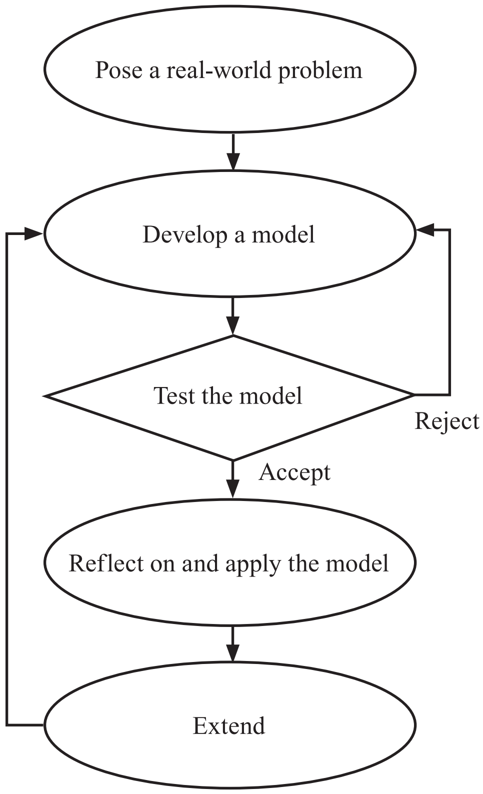

Figure 3

Technology

Technology is a powerful tool in the teaching and learning of mathematics. Technology can be used to enhance visualization and support student understanding of mathematical concepts. It can assist in the collection, recording, organization and analysis of data. Technology can increase the scope of the problem situations that are accessible to students. The use of technology increases the feasibility of students working with interesting problem contexts where students reflect, reason, solve problems and make decisions.

As teachers tie together the unifying themes of **mathematical inquiry**, **mathematical modelling and applications** and the **use of technology**, they should begin by providing substantial guidance, and then gradually encourage students to become more independent as inquirers and thinkers. IB students should learn to become strong communicators through the language of mathematics. Teachers should create a safe learning environment in which students are comfortable as risk-takers.

Teachers are encouraged to relate the mathematics being studied to other subjects and to the real world, especially topics that have particular relevance or are of interest to their students. Everyday problems and questions should be drawn into the lessons to motivate students and keep the material relevant; suggestions are provided in the “Links” column of the syllabus. The mathematical exploration offers an opportunity to investigate the usefulness, relevance and occurrence of mathematics in the real world and will add an extra dimension to the course. The emphasis is on communication by means of mathematical forms (for example, formulae, diagrams, graphs and so on) with accompanying commentary. Modelling, investigation, reflection, personal engagement and mathematical communication should therefore feature prominently in the DP mathematics classroom.

For further information on "Approaches to teaching a DP course", please refer to the publication *The Diploma Programme: From principles into practice* (April 2009). To support teachers, a variety of resources can be found on the OCC and details of workshops for professional development are available on the public website.

Format of the syllabus

- **Content:** this column lists, under each topic, the sub-topics to be covered.
- **Further guidance:** this column contains more detailed information on specific sub-topics listed in the content column. This clarifies the content for examinations.
- **Links:** this column provides useful links to the aims of the mathematics SL course, with suggestions for discussion, real-life examples and ideas for further investigation. **These suggestions are only a guide for introducing and illustrating the sub-topic and are not exhaustive.** Links are labelled as follows.

  Appl real-life examples and links to other DP subjects

  Aim 8 moral, social and ethical implications of the sub-topic

  Int international-mindedness

  TOK suggestions for discussion

Note that any syllabus references to other subject guides given in the "Links" column are correct for the current (2012) published versions of the guides.

Notes on the syllabus

- Formulae are only included in this document where there may be some ambiguity. All formulae required for the course are in the mathematics SL formula booklet.
- The term "technology" is used for any form of calculator or computer that may be available. However, there will be restrictions on which technology may be used in examinations, which will be noted in relevant documents.
- The terms "analysis" and "analytic approach" are generally used when referring to an approach that does not use technology.

Course of study

The content of all six topics in the syllabus must be taught, although not necessarily in the order in which they appear in this guide. Teachers are expected to construct a course of study that addresses the needs of their students and includes, where necessary, the topics noted in prior learning.

Integration of the mathematical exploration

Work leading to the completion of the exploration should be integrated into the course of study. Details of how to do this are given in the section on internal assessment and in the teacher support material.

Time allocation

The recommended teaching time for standard level courses is 150 hours. For mathematics SL, it is expected that 10 hours will be spent on work for the exploration. The time allocations given in this guide are approximate, and are intended to suggest how the remaining 140 hours allowed for the teaching of the syllabus might be allocated. However, the exact time spent on each topic depends on a number of factors, including the background knowledge and level of preparedness of each student. Teachers should therefore adjust these timings to correspond to the needs of their students.

Use of calculators

Students are expected to have access to a graphic display calculator (GDC) at all times during the course. The minimum requirements are reviewed as technology advances, and updated information will be provided to schools. It is expected that teachers and schools monitor calculator use with reference to the calculator policy. Regulations covering the types of calculators allowed in examinations are provided in the *Handbook of procedures for the Diploma Programme*. Further information and advice is provided in the *Mathematics HL/SL: Graphic display calculators teacher support material* (May 2005) and on the OCC.

Mathematics SL formula booklet

Each student is required to have access to a clean copy of this booklet during the examination. It is recommended that teachers ensure students are familiar with the contents of this document from the beginning of the course. It is the responsibility of the school to download a copy from IBIS or the OCC, check that there are no printing errors, and ensure that there are sufficient copies available for all students.

Teacher support materials

A variety of teacher support materials will accompany this guide. These materials will include guidance for teachers on the introduction, planning and marking of the exploration, and specimen examination papers and markschemes.

Command terms and notation list

Teachers and students need to be familiar with the IB notation and the command terms, as these will be used without explanation in the examination papers. The “Glossary of command terms” and “Notation list” appear as appendices in this guide.

Prior learning topics

As noted in the previous section on prior learning, it is expected that all students have extensive previous mathematical experiences, but these will vary. It is expected that mathematics SL students will be familiar with the following topics before they take the examinations, because questions assume knowledge of them. Teachers must therefore ensure that any topics listed here that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematics SL. This table lists the knowledge, together with the syllabus content, that is essential to successful completion of the mathematics SL course.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

Topic	Content
Number	Routine use of addition, subtraction, multiplication and division, using integers, decimals and fractions, including order of operations. Simple positive exponents. Simplification of expressions involving roots (surds or radicals). Prime numbers and factors, including greatest common divisors and least common multiples. Simple applications of ratio, percentage and proportion, linked to similarity. Definition and elementary treatment of absolute value (modulus), $\|a\|$. Rounding, decimal approximations and significant figures, including appreciation of errors. Expression of numbers in standard form (scientific notation), that is, $a \times 10^k$, $1 \le a < 10$, $k \in \mathbb{Z}$.
Sets and numbers	Concept and notation of sets, elements, universal (reference) set, empty (null) set, complement, subset, equality of sets, disjoint sets. Operations on sets: union and intersection. Commutative, associative and distributive properties. Venn diagrams. Number systems: natural numbers; integers, $\mathbb{Z}$; rationals, $\mathbb{Q}$, and irrationals; real numbers, $\mathbb{R}$. Intervals on the real number line using set notation and using inequalities. Expressing the solution set of a linear inequality on the number line and in set notation. Mappings of the elements of one set to another. Illustration by means of sets of ordered pairs, tables, diagrams and graphs.
Algebra	Manipulation of simple algebraic expressions involving factorization and expansion, including quadratic expressions. Rearrangement, evaluation and combination of simple formulae. Examples from other subject areas, particularly the sciences, should be included. The linear function and its graph, gradient and y-intercept. Addition and subtraction of algebraic fractions. The properties of order relations: $<, \le, >, \ge$. Solution of equations and inequalities in one variable, including cases with rational coefficients. Solution of simultaneous equations in two variables.

Topic	Content
Trigonometry	Angle measurement in degrees. Compass directions and three figure bearings. Right-angle trigonometry. Simple applications for solving triangles. Pythagoras’ theorem and its converse.
Geometry	Simple geometric transformations: translation, reflection, rotation, enlargement. Congruence and similarity, including the concept of scale factor of an enlargement. The circle, its centre and radius, area and circumference. The terms “arc”, “sector”, “chord”, “tangent” and “segment”. Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapeziums (trapezoids); compound shapes. Volumes of prisms, pyramids, spheres, cylinders and cones.
Coordinate geometry	Elementary geometry of the plane, including the concepts of dimension for point, line, plane and space. The equation of a line in the form $y = mx + c$. Parallel and perpendicular lines, including $m_1 = m_2$ and $m_1 m_2 = -1$. Geometry of simple plane figures. The Cartesian plane: ordered pairs (x, y), origin, axes. Mid-point of a line segment and distance between two points in the Cartesian plane and in three dimensions.
Statistics and probability	Descriptive statistics: collection of raw data; display of data in pictorial and diagrammatic forms, including pie charts, pictograms, stem and leaf diagrams, bar graphs and line graphs. Obtaining simple statistics from discrete and continuous data, including mean, median, mode, quartiles, range, interquartile range. Calculating probabilities of simple events.

Syllabus content

Topic 1—Algebra 9 hours

The aim of this topic is to introduce students to some basic algebraic concepts and applications.

	Content	Further guidance	Links
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications.	Technology may be used to generate and display sequences in several ways. Link to 2.6, exponential functions. Examples include compound interest and population growth.	**Int:** The chess legend (Sissa ibn Dahir). **Int:** Aryabhatta is sometimes considered the “father of algebra”. Compare with al-Khawarizmi. **TOK:** How did Gauss add up integers from 1 to 100? Discuss the idea of mathematical intuition as the basis for formal proof. **TOK:** Debate over the validity of the notion of “infinity”: finitists such as L. Kronecker consider that “a mathematical object does not exist unless it can be constructed from natural numbers in a finite number of steps”. **TOK:** What is Zeno’s dichotomy paradox? How far can mathematical facts be from intuition?

	Content	Further guidance	Links
1.2	Elementary treatment of exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	*Examples*: $16^{\frac{3}{4}} = 8$; $\frac{3}{4} = \log_{16} 8$; $\log 32 = 5 \log 2$; $(2^3)^{-4} = 2^{-12}$. *Examples*: $\log_4 7 = \frac{\ln 7}{\ln 4}$, $\log_{25} 125 = \frac{\log_5 125}{\log_5 25} \left(= \frac{3}{2} \right)$. Link to 2.6, logarithmic functions.	**Appl:** Chemistry 18.1 (Calculation of pH). **TOK:** Are logarithms an invention or discovery? (This topic is an opportunity for teachers to generate reflection on "the nature of mathematics".)
1.3	The binomial theorem: expansion of $(a+b)^n$, $n \in \mathbb{N}$. Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$. **Not required:** formal treatment of permutations and formula for ${}^nP_r$.	Counting principles may be used in the development of the theorem. $\binom{n}{r}$ should be found using **both** the formula and technology. *Example*: finding $\binom{6}{r}$ from inputting $y = 6^nC_rX$ and then reading coefficients from the table. Link to 5.8, binomial distribution.	**Aim 8:** Pascal's triangle. Attributing the origin of a mathematical discovery to the wrong mathematician. **Int:** The so-called "Pascal's triangle" was known in China much earlier than Pascal.

Topic 2—Functions and equations 24 hours

The aims of this topic are to explore the notion of a function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic, rather than elaborate analytical techniques. On examination papers, questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus, and students may need to choose the appropriate viewing window. For those functions explicitly mentioned, questions may also be set on composition of these functions with the linear function $y = ax + b$.

	Content	Further guidance	Links
2.1	Concept of function $f : x \mapsto f(x)$. Domain, range; image (value). Composite functions. Identity function. Inverse function f^{-1}. **Not required:** domain restriction.	*Example*: for $x \mapsto \sqrt{2-x}$, domain is $x \le 2$, range is $y \ge 0$. A graph is helpful in visualizing the range. $(f \circ g)(x) = f(g(x))$. $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$. On examination papers, students will only be asked to find the inverse of a *one-to-one* function.	**Int:** The development of functions, Rene Descartes (France), Gottfried Wilhelm Leibniz (Germany) and Leonhard Euler (Switzerland). **TOK:** Is zero the same as “nothing”? **TOK:** Is mathematics a formal language?
2.2	The graph of a function; its equation $y = f(x)$. Function graphing skills. Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range. Use of technology to graph a variety of functions, including ones not specifically mentioned. The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$.	**Note** the difference in the command terms “draw” and “sketch”. An analytic approach is also expected for simple functions, including all those listed under topic 2. Link to 6.3, local maximum and minimum points.	**Appl:** Chemistry 11.3.1 (sketching and interpreting graphs); geographic skills. **TOK:** How accurate is a visual representation of a mathematical concept? (Limits of graphs in delivering information about functions and phenomena in general, relevance of modes of representation.)

	Content	Further guidance	Links
2.3	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = pf(x)$. Stretch in the x-direction with scale factor $\frac{1}{q}$: $y = f\left(qx\right)$. Composite transformations.	Technology should be used to investigate these transformations. Translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ denotes horizontal shift of 3 units to the right, and vertical shift of 2 down. *Example*: $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the y-direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.	**Appl:** Economics 1.1 (shifting of supply and demand curves).
2.4	The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry. The form $x \mapsto a(x-p)(x-q)$, x-intercepts $(p, 0)$ and $(q, 0)$. The form $x \mapsto a(x-h)^2 + k$, vertex (h, k) .	Candidates are expected to be able to change from one form to another. Links to 2.3, transformations; 2.7, quadratic equations.	**Appl:** Chemistry 17.2 (equilibrium law). **Appl:** Physics 2.1 (kinematics). **Appl:** Physics 4.2 (simple harmonic motion). **Appl:** Physics 9.1 (HL only) (projectile motion).

	Content	Further guidance	Links
2.5	The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.		
	The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph.	*Examples*: $h(x) = \frac{4}{3x-2}$, $x \neq \frac{2}{3}$; $y = \frac{x+7}{2x-5}$, $x \neq \frac{5}{2}$.	
	Vertical and horizontal asymptotes.	Diagrams should include all asymptotes and intercepts.	
2.6	Exponential functions and their graphs: $x \mapsto a^x$, $a > 0$, $x \mapsto e^x$.		**Int:** The Babylonian method of multiplication: $ab = \frac{(a+b)^2 - a^2 - b^2}{2}$. Sulba Sutras in ancient India and the Bakhshali Manuscript contained an algebraic formula for solving quadratic equations.
	Logarithmic functions and their graphs: $x \mapsto \log_a x$, $x > 0$, $x \mapsto \ln x$, $x > 0$.	Links to 1.1, geometric sequences; 1.2, laws of exponents and logarithms; 2.1, inverse functions; 2.2, graphs of inverses; and 6.1, limits.	
	Relationships between these functions: $a^x = e^{x \ln a}$; $\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.		

	Content	Further guidance	Links
2.7	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Solving $ax^2+bx+c=0$, $a\neq 0$. The quadratic formula. The discriminant $\Delta=b^2-4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots. Solving exponential equations.	Solutions may be referred to as roots of equations or zeros of functions. Links to 2.2, function graphing skills; and 2.3–2.6, equations involving specific functions. *Examples*: $e^x=\sin x$, $x^4+5x-6=0$. *Example*: Find k given that the equation $3kx^2+2x+k=0$ has two equal real roots. *Examples*: $2^{x-1}=10$, $\left(\frac{1}{3}\right)^x=9^{x+1}$. Link to 1.2, exponents and logarithms.	
2.8	Applications of graphing skills and solving equations that relate to real-life situations.	Link to 1.1, geometric series.	**Appl:** Compound interest, growth and decay; projectile motion; braking distance; electrical circuits. **Appl:** Physics 7.2.7–7.2.9, 13.2.5, 13.2.6, 13.2.8 (radioactive decay and half-life)

Topic 3—Circular functions and trigonometry 16 hours

The aims of this topic are to explore the circular functions and to solve problems using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated.

	Content	Further guidance	Links
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as exact multiples of π, or decimals.	**Int:** Seki Takakazu calculating π to ten decimal places. **Int:** Hipparchus, Menelaus and Ptolemy. **Int:** Why are there 360 degrees in a complete turn? Links to Babylonian mathematics. **TOK:** Which is a better measure of angle: radian or degree? What are the "best" criteria by which to decide? **TOK:** Euclid's axioms as the building blocks of Euclidean geometry. Link to non-Euclidean geometry.
3.2	Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$. Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.	The equation of a straight line through the origin is $y = x\tan\theta$. *Examples*: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan 210° = \frac{\sqrt{3}}{3}$.	**Aim 8:** Who really invented "Pythagoras' theorem"? **Int:** The first work to refer explicitly to the sine as a function of an angle is the Aryabhatiya of Aryabhata (ca. 510). **TOK:** Trigonometry was developed by successive civilizations and cultures. How is mathematical knowledge considered from a sociocultural perspective?

	Content	Further guidance	Links
3.3	The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double angle identities for sine and cosine. Relationship between trigonometric ratios.	Simple geometrical diagrams and/or technology may be used to illustrate the double angle formulae (and other trigonometric identities). *Examples*: Given $\sin\theta$, finding possible values of $\tan\theta$ without finding θ. Given $\cos x = \frac{3}{4}$, and x is acute, find $\sin 2x$ without finding x.	
3.4	The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs. Composite functions of the form $f(x) = a\sin(b(x+c)) + d$. Transformations. Applications.	*Examples*: $f(x) = \tan\left(x - \frac{\pi}{4}\right)$, $f(x) = 2\cos(3(x-4)) + 1$. *Example*: $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the y-direction and a stretch of scale factor $\frac{1}{2}$ in the x-direction. Link to 2.3, transformation of graphs. Examples include height of tide, motion of a Ferris wheel.	**Appl:** Physics 4.2 (simple harmonic motion).

	Content	Further guidance	Links
3.5	Solving trigonometric equations in a finite interval, both graphically and analytically. Equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$. **Not required:** the general solution of trigonometric equations.	*Examples*: $2\sin x = 1$, $0 \le x \le 2\pi$, $2\sin 2x = 3\cos x$, $0^\circ \le x \le 180^\circ$, $2\tan\left(3(x-4)\right) = 1$, $-\pi \le x \le 3\pi$. *Examples*: $2\sin^2 x + 5\cos x + 1 = 0$ for $0 \le x < 4\pi$, $2\sin x = \cos 2x$, $-\pi \le x \le \pi$.	
3.6	Solution of triangles. The cosine rule. The sine rule, including the ambiguous case. Area of a triangle, $\frac{1}{2}ab\sin C$. Applications.	Pythagoras' theorem is a special case of the cosine rule. Link with 4.2, scalar product, noting that: $\boldsymbol{c} = \boldsymbol{a} - \boldsymbol{b} \Rightarrow \lvert\boldsymbol{c}\rvert^2 = \lvert\boldsymbol{a}\rvert^2 + \lvert\boldsymbol{b}\rvert^2 - 2\boldsymbol{a}\cdot\boldsymbol{b}$. Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.	**Aim 8:** Attributing the origin of a mathematical discovery to the wrong mathematician. **Int:** Cosine rule: Al-Kashi and Pythagoras. **TOK:** Non-Euclidean geometry: angle sum on a globe greater than 180°.

Topic 4—Vectors

16 hours

The aim of this topic is to provide an elementary introduction to vectors, including both algebraic and geometric approaches. The use of dynamic geometry software is extremely helpful to visualize situations in three dimensions.

	Content	Further guidance	Links
4.1	Vectors as displacements in the plane and in three dimensions.	Link to three-dimensional geometry, x, y and z-axes.	**Appl:** Physics 1.3.2 (vector sums and differences) Physics 2.2.2, 2.2.3 (vector resultants). **TOK:** How do we relate a theory to the author? Who developed vector analysis: JW Gibbs or O Heaviside?
	Components of a vector; column representation; $\boldsymbol{v}=\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}=v_1\boldsymbol{i}+v_2\boldsymbol{j}+v_3\boldsymbol{k}$.	Components are with respect to the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ (standard basis).	
	Algebraic and geometric approaches to the following:	Applications to simple geometric figures are essential.	
	• the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;	The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v}-\boldsymbol{w}=\boldsymbol{v}+(-\boldsymbol{w})$. Vector sums and differences can be represented by the diagonals of a parallelogram.	
	• multiplication by a scalar, $k\boldsymbol{v}$; parallel vectors;	Multiplication by a scalar can be illustrated by enlargement.	
	• magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;		
	• unit vectors; base vectors; $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$;		
	• position vectors $\overrightarrow{\mathrm{OA}}=\boldsymbol{a}$;		
	• $\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{OB}}-\overrightarrow{\mathrm{OA}}=\boldsymbol{b}-\boldsymbol{a}$.	Distance between points A and B is the magnitude of $\overrightarrow{\mathrm{AB}}$.	

	Content	Further guidance	Links
4.2	The scalar product of two vectors.	The scalar product is also known as the “dot product”. Link to 3.6, cosine rule.	
	Perpendicular vectors; parallel vectors.	For non-zero vectors, $\boldsymbol{v} \cdot \boldsymbol{w} = 0$ is equivalent to the vectors being perpendicular. For parallel vectors, $\boldsymbol{w} = k\boldsymbol{v}$, $\lvert \boldsymbol{v} \cdot \boldsymbol{w} \rvert = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert$.	
	The angle between two vectors.		
4.3	Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.	Relevance of $\boldsymbol{a}$ (position) and $\boldsymbol{b}$ (direction). Interpretation of t as time and $\boldsymbol{b}$ as velocity, with $\lvert \boldsymbol{b} \rvert$ representing speed.	**Aim 8:** Vector theory is used for tracking displacement of objects, including for peaceful and harmful purposes. **TOK:** Are algebra and geometry two separate domains of knowledge? (Vector algebra is a good opportunity to discuss how geometrical properties are described and generalized by algebraic methods.)
	The angle between two lines.		
4.4	Distinguishing between coincident and parallel lines. Finding the point of intersection of two lines. Determining whether two lines intersect.		

Topic 5—Statistics and probability 35 hours

The aim of this topic is to introduce basic concepts. It is expected that most of the calculations required will be done using technology, but explanations of calculations by hand may enhance understanding. The emphasis is on understanding and interpreting the results obtained, in context. Statistical tables will no longer be allowed in examinations. While many of the calculations required in examinations are estimates, it is likely that the command terms “write down”, “find” and “calculate” will be used.

	Content	Further guidance	Links
5.1	Concepts of population, sample, random sample, discrete and continuous data.	Continuous and discrete data.	**Appl:** Psychology: descriptive statistics, random sample (various places in the guide).
	Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals;		**Aim 8:** Misleading statistics. **Int:** The St Petersburg paradox, Chebychev, Pavlovsky.
	box-and-whisker plots; outliers.	Outlier is defined as more than $1.5 \times \text{IQR}$ from the nearest quartile. Technology may be used to produce histograms and box-and-whisker plots.	
	Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class. **Not required:** frequency density histograms.		

	Content	Further guidance	Links
5.2	Statistical measures and their interpretations. Central tendency: mean, median, mode. Quartiles, percentiles. Dispersion: range, interquartile range, variance, standard deviation. Effect of constant changes to the original data. Applications.	On examination papers, data will be treated as the population. Calculation of mean using formula and technology. Students should use mid-interval values to estimate the mean of grouped data. Calculation of standard deviation/variance using only technology. Link to 2.3, transformations. *Examples*: If 5 is subtracted from all the data items, then the mean is decreased by 5, but the standard deviation is unchanged. If all the data items are doubled, the median is doubled, but the variance is increased by a factor of 4.	**Appl:** Psychology: descriptive statistics (various places in the guide). **Appl:** Statistical calculations to show patterns and changes; geographic skills; statistical graphs. **Appl:** Biology 1.1.2 (calculating mean and standard deviation); Biology 1.1.4 (comparing means and spreads between two or more samples). **Int:** Discussion of the different formulae for variance. **TOK:** Do different measures of central tendency express different properties of the data? Are these measures invented or discovered? Could mathematics make alternative, equally true, formulae? What does this tell us about mathematical truths? **TOK:** How easy is it to lie with statistics?
5.3	Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.	Values of the median and quartiles produced by technology may be different from those obtained from a cumulative frequency graph.	

	Content	Further guidance	Links
5.4	Linear correlation of bivariate data. Pearson's product–moment correlation coefficient r. Scatter diagrams; lines of best fit. Equation of the regression line of y on x. Use of the equation for prediction purposes. Mathematical and contextual interpretation. **Not required:** the coefficient of determination R^2.	Independent variable x, dependent variable y. Technology should be used to calculate r. However, hand calculations of r may enhance understanding. Positive, zero, negative; strong, weak, no correlation. The line of best fit passes through the mean point. Technology should be used find the equation. Interpolation, extrapolation.	**Appl:** Chemistry 11.3.3 (curves of best fit). **Appl:** Geography (geographic skills). Measures of correlation; geographic skills. **Appl:** Biology 1.1.6 (correlation does not imply causation). **TOK:** Can we predict the value of x from y, using this equation? **TOK:** Can all data be modelled by a (known) mathematical function? Consider the reliability and validity of mathematical models in describing real-life phenomena.
5.5	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A is $\mathrm{P}(A)=\frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Use of Venn diagrams, tree diagrams and tables of outcomes.	The sample space can be represented diagrammatically in many ways. Experiments using coins, dice, cards and so on, can enhance understanding of the distinction between (experimental) relative frequency and (theoretical) probability. Simulations may be used to enhance this topic. Links to 5.1, frequency; 5.3, cumulative frequency.	**TOK:** To what extent does mathematics offer models of real life? Is there always a function to model data behaviour?

	Content	Further guidance	Links
5.6	Combined events, $P(A \cup B)$. Mutually exclusive events: $P(A \cap B) = 0$. Conditional probability; the definition $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. Independent events; the definition $P(A \mid B) = P(A) = P(A \mid B')$. Probabilities with and without replacement.	The non-exclusivity of "or". Problems are often best solved with the aid of a Venn diagram or tree diagram, without explicit use of formulae.	**Aim 8:** The gambling issue: use of probability in casinos. Could or should mathematics help increase incomes in gambling? **TOK:** Is mathematics useful to measure risks? **TOK:** Can gambling be considered as an application of mathematics? (This is a good opportunity to generate a debate on the nature, role and ethics of mathematics regarding its applications.)
5.7	Concept of discrete random variables and their probability distributions. Expected value (mean), $E(X)$ for discrete data. Applications.	Simple examples only, such as: $P(X = x) = \frac{1}{18}(4 + x)$ for $x \in \{1, 2, 3\}$; $P(X = x) = \frac{5}{18}, \frac{6}{18}, \frac{7}{18}$. $E(X) = 0$ indicates a fair game where X represents the gain of one of the players. Examples include games of chance.	

	Content	Further guidance	Links
5.8	Binomial distribution. Mean and variance of the binomial distribution. **Not required:** formal proof of mean and variance.	Link to 1.3, binomial theorem. Conditions under which random variables have this distribution. Technology is usually the best way of calculating binomial probabilities.	
5.9	Normal distributions and curves. Standardization of normal variables (z-values, z-scores). Properties of the normal distribution.	Probabilities and values of the variable must be found using technology. Link to 2.3, transformations. The standardized value (z) gives the number of standard deviations from the mean.	**Appl:** Biology 1.1.3 (links to normal distribution). **Appl:** Psychology: descriptive statistics (various places in the guide).

Topic 6—Calculus

40 hours

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their applications.

	Content	Further guidance	Links
6.1	Informal ideas of limit and convergence.	*Example*: 0.3, 0.33, 0.333, ... converges to $\frac{1}{3}$. Technology should be used to explore ideas of limits, numerically and graphically.	**Appl:** Economics 1.5 (marginal cost, marginal revenue, marginal profit). **Appl:** Chemistry 11.3.4 (interpreting the gradient of a curve). **Aim 8:** The debate over whether Newton or Leibnitz discovered certain calculus concepts. **TOK:** What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life? **TOK:** Opportunities for discussing hypothesis formation and testing, and then the formal proof can be tackled by comparing certain cases, through an investigative approach.
	Limit notation.	*Example*: $\lim_{x\to\infty}\left(\frac{2x+3}{x-1}\right)$ Links to 1.1, infinite geometric series; 2.5–2.7, rational and exponential functions, and asymptotes.	
	Definition of derivative from first principles as $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.	Use of this definition for derivatives of simple polynomial functions only. Technology could be used to illustrate other derivatives. Link to 1.3, binomial theorem. Use of both forms of notation, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $f'(x)$, for the first derivative.	
	Derivative interpreted as gradient function and as rate of change.	Identifying intervals on which functions are increasing or decreasing.	
	Tangents and normals, and their equations. **Not required:** analytic methods of calculating limits.	Use of both analytic approaches and technology. Technology can be used to explore graphs and their derivatives.	

	Content	Further guidance	Links
6.2	Derivative of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.		
	Differentiation of a sum and a real multiple of these functions.		
	The chain rule for composite functions.	Link to 2.1, composition of functions.	
	The product and quotient rules.	Technology may be used to investigate the chain rule.	
	The second derivative.	Use of both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$.	
	Extension to higher derivatives.	$\frac{d^ny}{dx^n}$ and $f^{(n)}(x)$.	

	Content	Further guidance	Links
6.3	Local maximum and minimum points. Testing for maximum or minimum.	Using change of sign of the first derivative and using sign of the second derivative. Use of the terms "concave-up" for $f''(x) > 0$, and "concave-down" for $f''(x) < 0$.	**Appl:** profit, area, volume.
	Points of inflexion with zero and non-zero gradients.	At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.	
	Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''. Optimization.	Both "global" (for large $\lvert x \rvert$) and "local" behaviour. Technology can display the graph of a derivative without explicitly finding an expression for the derivative. Use of the first or second derivative test to justify maximum and/or minimum values.	
	Applications. **Not required:** points of inflexion where $f''(x)$ is not defined: for example, $y = x^{1/3}$ at $(0,0)$.	Examples include profit, area, volume. Link to 2.2, graphing functions.	

	Content	Further guidance	Links
6.4	Indefinite integration as anti-differentiation. Indefinite integral of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x.	$\int \frac{1}{x}\mathrm{d}x = \ln x + C$, $x > 0$.	
	The composites of any of these with the linear function $ax+b$.	*Example*: $f'(x) = \cos(2x+3) \Rightarrow f(x) = \frac{1}{2}\sin(2x+3) + C$.	
	Integration by inspection, or substitution of the form $\int f(g(x))g'(x)\mathrm{d}x$.	*Examples*: $\int 2x\left(x^2+1\right)^4 \mathrm{d}x$, $\int x\sin x^2 \mathrm{d}x$, $\int \frac{\sin x}{\cos x}\mathrm{d}x$.	
6.5	Anti-differentiation with a boundary condition to determine the constant term.	*Example*: if $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + x$ and $y = 10$ when $x = 0$, then $y = x^3 + \frac{1}{2}x^2 + 10$.	**Int:** Successful calculation of the volume of the pyramidal frustum by ancient Egyptians (Egyptian Moscow papyrus). Use of infinitesimals by Greek geometers.
	Definite integrals, both analytically and using technology.	$\int_a^b g'(x)\mathrm{d}x = g(b) - g(a)$. The value of some definite integrals can only be found using technology.	Accurate calculation of the volume of a cylinder by Chinese mathematician Liu Hui
	Areas under curves (between the curve and the x-axis). Areas between curves. Volumes of revolution about the x-axis.	Students are expected to first write a correct expression before calculating the area. Technology may be used to enhance understanding of area and volume.	**Int:** Ibn Al Haytham: first mathematician to calculate the integral of a function, in order to find the volume of a paraboloid.
6.6	Kinematic problems involving displacement s, velocity v and acceleration a.	$v = \frac{\mathrm{d}s}{\mathrm{d}t}$; $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$.	**Appl:** Physics 2.1 (kinematics).
	Total distance travelled.	Total distance travelled $= \int_{t_1}^{t_2} \lvert v \rvert \,\mathrm{d}t$.	

Assessment in the Diploma Programme

General

Assessment is an integral part of teaching and learning. The most important aims of assessment in the Diploma Programme are that it should support curricular goals and encourage appropriate student learning. Both external and internal assessment are used in the Diploma Programme. IB examiners mark work produced for external assessment, while work produced for internal assessment is marked by teachers and externally moderated by the IB.

There are two types of assessment identified by the IB.

- Formative assessment informs both teaching and learning. It is concerned with providing accurate and helpful feedback to students and teachers on the kind of learning taking place and the nature of students' strengths and weaknesses in order to help develop students' understanding and capabilities. Formative assessment can also help to improve teaching quality, as it can provide information to monitor progress towards meeting the course aims and objectives.
- Summative assessment gives an overview of previous learning and is concerned with measuring student achievement.

The Diploma Programme primarily focuses on summative assessment designed to record student achievement at or towards the end of the course of study. However, many of the assessment instruments can also be used formatively during the course of teaching and learning, and teachers are encouraged to do this. A comprehensive assessment plan is viewed as being integral with teaching, learning and course organization. For further information, see the IB *Programme standards and practices* document.

The approach to assessment used by the IB is criterion-related, not norm-referenced. This approach to assessment judges students' work by their performance in relation to identified levels of attainment, and not in relation to the work of other students. For further information on assessment within the Diploma Programme, please refer to the publication *Diploma Programme assessment: Principles and practice*.

To support teachers in the planning, delivery and assessment of the Diploma Programme courses, a variety of resources can be found on the OCC or purchased from the IB store (http://store.ibo.org). Teacher support materials, subject reports, internal assessment guidance, grade descriptors, as well as resources from other teachers, can be found on the OCC. Specimen and past examination papers as well as markschemes can be purchased from the IB store.

Methods of assessment

The IB uses several methods to assess work produced by students.

Assessment criteria

Assessment criteria are used when the assessment task is open-ended. Each criterion concentrates on a particular skill that students are expected to demonstrate. An assessment objective describes what students should be able to do, and assessment criteria describe how well they should be able to do it. Using assessment criteria allows discrimination between different answers and encourages a variety of responses. Each criterion comprises a set of hierarchically ordered level descriptors. Each level descriptor is worth one or more marks. Each criterion is applied independently using a best-fit model. The maximum marks for each criterion may differ according to the criterion's importance. The marks awarded for each criterion are added together to give the total mark for the piece of work.

Markbands

Markbands are a comprehensive statement of expected performance against which responses are judged. They represent a single holistic criterion divided into level descriptors. Each level descriptor corresponds to a range of marks to differentiate student performance. A best-fit approach is used to ascertain which particular mark to use from the possible range for each level descriptor.

Markschemes

This generic term is used to describe analytic markschemes that are prepared for specific examination papers. Analytic markschemes are prepared for those examination questions that expect a particular kind of response and/or a given final answer from the students. They give detailed instructions to examiners on how to break down the total mark for each question for different parts of the response. A markscheme may include the content expected in the responses to questions or may be a series of marking notes giving guidance on how to apply criteria.

Assessment outline

First examinations 2014

Assessment component	Weighting
External assessment (3 hours)	**80%**
Paper 1 (1 hour 30 minutes) No calculator allowed. (90 marks) **Section A** Compulsory short-response questions based on the whole syllabus. **Section B** Compulsory extended-response questions based on the whole syllabus.	**40%**
Paper 2 (1 hour 30 minutes) Graphic display calculator required. (90 marks) **Section A** Compulsory short-response questions based on the whole syllabus. **Section B** Compulsory extended-response questions based on the whole syllabus.	**40%**
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics SL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	**20%**

External assessment

General

Markschemes are used to assess students in both papers. The markschemes are specific to each examination.

External assessment details

Paper 1 and paper 2

These papers are externally set and externally marked. Together, they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.

Calculators

Paper 1

Students are not permitted access to any calculator. Questions will mainly involve analytic approaches to solutions, rather than requiring the use of a GDC. The paper is not intended to require complicated calculations, with the potential for careless errors. However, questions will include some arithmetical manipulations when they are essential to the development of the question.

Paper 2

Students must have access to a GDC at all times. However, not all questions will necessarily require the use of the GDC. Regulations covering the types of GDC allowed are provided in the *Handbook of procedures for the Diploma Programme*.

Mathematics SL formula booklet

Each student must have access to a clean copy of the formula booklet during the examination. It is the responsibility of the school to download a copy from IBIS or the OCC and to ensure that there are sufficient copies available for all students.

Awarding of marks

Marks may be awarded for method, accuracy, answers and reasoning, including interpretation.

In paper 1 and paper 2, full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations (in the form of, for example, diagrams, graphs or calculations). Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. All students should therefore be advised to show their working.

Paper 1

Duration: 1 hour 30 minutes
Weighting: 40%

- This paper consists of section A, short-response questions, and section B, extended-response questions.
- Students are not permitted access to any calculator on this paper.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.

Mark allocation

- This paper is worth **90** marks, representing **40%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily each be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Section A

This section consists of compulsory short-response questions based on the whole syllabus. It is worth approximately 45 marks.

The intention of this section is to test students' knowledge and understanding across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps is needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Section B

This section consists of a small number of compulsory extended-response questions based on the whole syllabus. It is worth approximately 45 marks. Individual questions may require knowledge of more than one topic.

The intention of this section is to test students' knowledge and understanding of the syllabus in depth. The range of syllabus topics tested in this section may be narrower than that tested in section A.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions will develop a single theme.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem-solving.

Paper 2

Duration: 1 hour 30 minutes
Weighting: 40%

This paper consists of section A, short-response questions, and section B, extended-response questions. A GDC is required for this paper, but not every question will necessarily require its use.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.

Mark allocation

- This paper is worth **90** marks, representing **40%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily each be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Section A

This section consists of compulsory short-response questions based on the whole syllabus. It is worth approximately 45 marks.

The intention of this section is to test students' knowledge and understanding across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps is needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Section B

This section consists of a small number of compulsory extended-response questions based on the whole syllabus. It is worth approximately 45 marks. Individual questions may require knowledge of more than one topic.

The intention of this section is to test students' knowledge and understanding of the syllabus in depth. The range of syllabus topics tested in this section may be narrower than that tested in section A.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions will develop a single theme.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem-solving.

Internal assessment

Purpose of internal assessment

Internal assessment is an integral part of the course and is compulsory for all students. It enables students to demonstrate the application of their skills and knowledge, and to pursue their personal interests, without the time limitations and other constraints that are associated with written examinations. The internal assessment should, as far as possible, be woven into normal classroom teaching and not be a separate activity conducted after a course has been taught.

Internal assessment in mathematics SL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. It is marked according to five assessment criteria.

Guidance and authenticity

The exploration submitted for internal assessment must be the student's own work. However, it is not the intention that students should decide upon a title or topic and be left to work on the exploration without any further support from the teacher. The teacher should play an important role during both the planning stage and the period when the student is working on the exploration. It is the responsibility of the teacher to ensure that students are familiar with:

- the requirements of the type of work to be internally assessed
- the IB academic honesty policy available on the OCC
- the assessment criteria—students must understand that the work submitted for assessment must address these criteria effectively.

Teachers and students must discuss the exploration. Students should be encouraged to initiate discussions with the teacher to obtain advice and information, and students must not be penalized for seeking guidance. However, if a student could not have completed the exploration without substantial support from the teacher, this should be recorded on the appropriate form from the *Handbook of procedures for the Diploma Programme.*

It is the responsibility of teachers to ensure that all students understand the basic meaning and significance of concepts that relate to academic honesty, especially authenticity and intellectual property. Teachers must ensure that all student work for assessment is prepared according to the requirements and must explain clearly to students that the exploration must be entirely their own.

As part of the learning process, teachers can give advice to students on a **first draft** of the exploration. This advice should be in terms of the way the work could be improved, but this first draft must not be heavily annotated or edited by the teacher. The next version handed to the teacher after the first draft must be the final one.

All work submitted to the IB for moderation or assessment must be authenticated by a teacher, and must not include any known instances of suspected or confirmed malpractice. Each student must sign the coversheet for internal assessment to confirm that the work is his or her authentic work and constitutes the final version of that work. Once a student has officially submitted the final version of the work to a teacher (or the coordinator) for internal assessment, together with the signed coversheet, it cannot be retracted.

Authenticity may be checked by discussion with the student on the content of the work, and scrutiny of one or more of the following:

- the student's initial proposal
- the first draft of the written work
- the references cited
- the style of writing compared with work known to be that of the student.

The requirement for teachers and students to sign the coversheet for internal assessment applies to the work of all students, not just the sample work that will be submitted to an examiner for the purpose of moderation. If the teacher and student sign a coversheet, but there is a comment to the effect that the work may not be authentic, the student will not be eligible for a mark in that component and no grade will be awarded. For further details refer to the IB publication *Academic honesty* and the relevant articles in the *General regulations: Diploma Programme*.

The same piece of work cannot be submitted to meet the requirements of both the internal assessment and the extended essay.

Group work

Group work should not be used for explorations. Each exploration is an individual piece of work.

It should be made clear to students that all work connected with the exploration, including the writing of the exploration, should be their own. It is therefore helpful if teachers try to encourage in students a sense of responsibility for their own learning so that they accept a degree of ownership and take pride in their own work.

Time allocation

Internal assessment is an integral part of the mathematics SL course, contributing 20% to the final assessment in the course. This weighting should be reflected in the time that is allocated to teaching the knowledge, skills and understanding required to undertake the work as well as the total time allocated to carry out the work.

It is expected that a total of approximately 10 teaching hours should be allocated to the work. This should include:

- time for the teacher to explain to students the requirements of the exploration
- class time for students to work on the exploration
- time for consultation between the teacher and each student
- time to review and monitor progress, and to check authenticity.

Using assessment criteria for internal assessment

For internal assessment, a number of assessment criteria have been identified. Each assessment criterion has level descriptors describing specific levels of achievement together with an appropriate range of marks. The level descriptors concentrate on positive achievement, although for the lower levels failure to achieve may be included in the description.

Teachers must judge the internally assessed work against the criteria using the level descriptors.

- The aim is to find, for each criterion, the descriptor that conveys most accurately the level attained by the student.
- When assessing a student's work, teachers should read the level descriptors for each criterion, starting with level 0, until they reach a descriptor that describes a level of achievement that has not been reached. The level of achievement gained by the student is therefore the preceding one, and it is this that should be recorded.
- Only whole numbers should be recorded; partial marks, that is fractions and decimals, are not acceptable.
- Teachers should not think in terms of a pass or fail boundary, but should concentrate on identifying the appropriate descriptor for each assessment criterion.
- The highest level descriptors do not imply faultless performance but should be achievable by a student. Teachers should not hesitate to use the extremes if they are appropriate descriptions of the work being assessed.
- A student who attains a high level of achievement in relation to one criterion will not necessarily attain high levels of achievement in relation to the other criteria. Similarly, a student who attains a low level of achievement for one criterion will not necessarily attain low achievement levels for the other criteria. Teachers should not assume that the overall assessment of the students will produce any particular distribution of marks.
- It is expected that the assessment criteria be made available to students.

Internal assessment details

Mathematical exploration

Duration: 10 teaching hours
Weighting: 20%

Introduction

The internally assessed component in this course is a mathematical exploration. This is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs and so on), with accompanying commentary, good mathematical writing and thoughtful reflection. A student should develop his or her own focus, with the teacher providing feedback via, for example, discussion and interview. This will allow the students to develop area(s) of interest to them without a time constraint as in an examination, and allow all students to experience a feeling of success.

The final report should be approximately 6 to 12 pages long. It can be either word processed or handwritten. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it fairly easily. The report should include a detailed bibliography, and sources need to be referenced in line with the IB academic honesty policy. Direct quotes must be acknowledged.

The purpose of the exploration

The aims of the mathematics SL course are carried through into the objectives that are formally assessed as part of the course, through either written examination papers, or the exploration, or both. In addition to testing the objectives of the course, the exploration is intended to provide students with opportunities to increase their understanding of mathematical concepts and processes, and to develop a wider appreciation of mathematics. These are noted in the aims of the course, **in particular, aims 6–9 (applications, technology, moral, social**

and ethical implications, and the international dimension). It is intended that, by doing the exploration, students benefit from the mathematical activities undertaken and find them both stimulating and rewarding. It will enable students to acquire the attributes of the IB learner profile.

The specific purposes of the exploration are to:

- develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- provide opportunities for students to complete a piece of mathematical work over an extended period of time
- enable students to experience the satisfaction of applying mathematical processes independently
- provide students with the opportunity to experience for themselves the beauty, power and usefulness of mathematics
- encourage students, where appropriate, to discover, use and appreciate the power of technology as a mathematical tool
- enable students to develop the qualities of patience and persistence, and to reflect on the significance of their work
- provide opportunities for students to show, with confidence, how they have developed mathematically.

Management of the exploration

Work for the exploration should be incorporated into the course so that students are given the opportunity to learn the skills needed. Time in class can therefore be used for general discussion of areas of study, as well as familiarizing students with the criteria.

Further details on the development of the exploration are included in the teacher support material.

Requirements and recommendations

Students can choose from a wide variety of activities, for example, modelling, investigations and applications of mathematics. To assist teachers and students in the choice of a topic, a list of stimuli is available in the teacher support material. However, students are not restricted to this list.

The exploration should not normally exceed 12 pages, including diagrams and graphs, but excluding the bibliography. However, it is the quality of the mathematical writing that is important, not the length.

The teacher is expected to give appropriate guidance at all stages of the exploration by, for example, directing students into more productive routes of inquiry, making suggestions for suitable sources of information, and providing advice on the content and clarity of the exploration in the writing-up stage.

Teachers are responsible for indicating to students the existence of errors but should not explicitly correct these errors. It must be emphasized that students are expected to consult the teacher throughout the process.

All students should be familiar with the requirements of the exploration and the criteria by which it is assessed. Students need to start planning their explorations as early as possible in the course. Deadlines should be firmly established. There should be a date for submission of the exploration topic and a brief outline description, a date for the submission of the first draft and, of course, a date for completion.

In developing their explorations, students should aim to make use of mathematics learned as part of the course. The mathematics used should be commensurate with the level of the course, that is, it should be similar to that suggested by the syllabus. It is not expected that students produce work that is outside the mathematics SL syllabus—however, this is not penalized.

Internal assessment criteria

The exploration is internally assessed by the teacher and externally moderated by the IB using assessment criteria that relate to the objectives for mathematics SL.

Each exploration is assessed against the following five criteria. The final mark for each exploration is the sum of the scores for each criterion. The maximum possible final mark is 20.

Students will not receive a grade for mathematics SL if they have not submitted an exploration.

Criterion A	Communication
Criterion B	Mathematical presentation
Criterion C	Personal engagement
Criterion D	Reflection
Criterion E	Use of mathematics

Criterion A: Communication

This criterion assesses the organization and coherence of the exploration. A well-organized exploration includes an introduction, has a rationale (which includes explaining why this topic was chosen), describes the aim of the exploration and has a conclusion. A coherent exploration is logically developed and easy to follow.

Graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, concise and complete.

Criterion B: Mathematical presentation

This criterion assesses to what extent the student is able to:

- use appropriate mathematical language (notation, symbols, terminology)
- define key terms, where required
- use multiple forms of mathematical representation, such as formulae, diagrams, tables, charts, graphs and models, where appropriate.

Students are expected to use mathematical language when communicating mathematical ideas, reasoning and findings.

Students are encouraged to choose and use appropriate ICT tools such as graphic display calculators, screenshots, graphing, spreadsheets, databases, drawing and word-processing software, as appropriate, to enhance mathematical communication.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is some appropriate mathematical presentation.
2	The mathematical presentation is mostly appropriate.
3	The mathematical presentation is appropriate throughout.

Criterion C: Personal engagement

This criterion assesses the extent to which the student engages with the exploration and makes it their own. Personal engagement may be recognized in different attributes and skills. These include thinking independently and/or creatively, addressing personal interest and presenting mathematical ideas in their own way.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited or superficial personal engagement.
2	There is evidence of some personal engagement.
3	There is evidence of significant personal engagement.
4	There is abundant evidence of outstanding personal engagement.

Criterion D: Reflection

This criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited or superficial reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

Criterion E: Use of mathematics

This criterion assesses to what extent students use mathematics in the exploration.

Students are expected to produce work that is commensurate with the level of the course. The mathematics explored should either be part of the syllabus, or at a similar level or beyond. It should not be completely based on mathematics listed in the prior learning. If the level of mathematics is not commensurate with the level of the course, a maximum of two marks can be awarded for this criterion.

The mathematics can be regarded as correct even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used.
2	Some relevant mathematics is used. Limited understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.

Glossary of command terms

Command terms with definitions

Students should be familiar with the following key terms and phrases used in examination questions, which are to be understood as described below. Although these terms will be used in examination questions, other terms may be used to direct students to present an argument in a specific way.

Calculate	Obtain a numerical answer showing the relevant stages in the working.
Comment	Give a judgment based on a given statement or result of a calculation.
Compare	Give an account of the similarities between two (or more) items or situations, referring to both (all) of them throughout.
Compare and contrast	Give an account of the similarities and differences between two (or more) items or situations, referring to both (all) of them throughout.
Construct	Display information in a diagrammatic or logical form.
Contrast	Give an account of the differences between two (or more) items or situations, referring to both (all) of them throughout.
Deduce	Reach a conclusion from the information given.
Demonstrate	Make clear by reasoning or evidence, illustrating with examples or practical application.
Describe	Give a detailed account.
Determine	Obtain the only possible answer.
Differentiate	Obtain the derivative of a function.
Distinguish	Make clear the differences between two or more concepts or items.
Draw	Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Estimate	Obtain an approximate value.
Explain	Give a detailed account, including reasons or causes.
Find	Obtain an answer, showing relevant stages in the working.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Identify	Provide an answer from a number of possibilities.

Integrate	Obtain the integral of a function.
Interpret	Use knowledge and understanding to recognize trends and draw conclusions from given information.
Investigate	Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions.
Justify	Give valid reasons or evidence to support an answer or conclusion.
Label	Add labels to a diagram.
List	Give a sequence of brief answers with no explanation.
Plot	Mark the position of points on a diagram.
Predict	Give an expected result.
Show	Give the steps in a calculation or derivation.
Show that	Obtain the required result (possibly using information given) without the formality of proof. “Show that” questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Verify	Provide evidence that validates the result.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

Notation list

Of the various notations in use, the IB has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listed in this guide are used on a particular examination paper, they are defined within the question in which they appear.

Because students are required to recognize, though not necessarily use, IB notation in examinations, it is recommended that teachers introduce students to this notation at the earliest opportunity. Students are **not** allowed access to information about this notation in the examinations.

Students must always use correct mathematical notation, not calculator notation.

$\mathbb{N}$	the set of positive integers and zero, $\{0, 1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$n(A)$	the number of elements in the finite set A
$\{x \mid \;\}$	the set of all x such that
$\in$	is an element of
$\notin$	is not an element of
$\varnothing$	the empty (null) set
U	the universal set
$\cup$	Union

$\cap$	Intersection
$\subset$	is a proper subset of
$\subseteq$	is a subset of
A'	the complement of the set A
$a \mid b$	a divides b
$a^{1/n}$, $\sqrt[n]{a}$	a to the power of $\frac{1}{n}$, n^{th} root of a (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$\lvert x \rvert$	modulus or absolute value of x, that is $\begin{cases} x & \text{for } x \geq 0, x \in \mathbb{R} \\ -x & \text{for } x < 0, x \in \mathbb{R} \end{cases}$
$\approx$	is approximately equal to
$>$	is greater than
$\geq$	is greater than or equal to
$<$	is less than
$\leq$	is less than or equal to
$\ngtr$	is not greater than
$\nless$	is not less than
u_n	the n^{th} term of a sequence or series
d	the common difference of an arithmetic sequence
r	the common ratio of a geometric sequence
S_n	the sum of the first n terms of a sequence, $u_1 + u_2 + ... + u_n$
S_∞	the sum to infinity of a sequence, $u_1 + u_2 + ...$
$\sum_{i=1}^{n} u_i$	$u_1 + u_2 + ... + u_n$
$\binom{n}{r}$	the r^{th} binomial coefficient, $r = 0, 1, 2, \ldots$, in the expansion of $(a+b)^n$
$n!$	$n(n-1)(n-2) \times \ldots \times 3 \times 2 \times 1$
$f : x \mapsto y$	f is a function under which x is mapped to y

$f(x)$	the image of x under the function f
f^{-1}	the inverse function of the function f
$f \circ g$	the composite function of f and g
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$f'(x)$	the derivative of $f(x)$ with respect to x
$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	the second derivative of y with respect to x
$f''(x)$	the second derivative of $f(x)$ with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the n^{th} derivative of y with respect to x
$f^{(n)}(x)$	the n^{th} derivative of $f(x)$ with respect to x
$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
e^x	exponential function (base e) of x
$\log_a x$	logarithm to the base a of x
$\ln x$	the natural logarithm of x, $\log_{\mathrm{e}} x$
sin, cos, tan	the circular functions
$\mathrm{A}(x, y)$	the point A in the plane with Cartesian coordinates x and y
[AB]	the line segment with end points A and B
AB	the length of [AB]
(AB)	the line containing points A and B
$\hat{A}$	the angle at A
$\mathrm{C\hat{A}B}$	the angle between [CA] and [AB]

$\Delta\mathrm{ABC}$	the triangle whose vertices are A, B and C
$\boldsymbol{v}$	the vector $\boldsymbol{v}$
$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment from A to B
$\boldsymbol{a}$	the position vector $\overrightarrow{\mathrm{OA}}$
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	unit vectors in the directions of the Cartesian coordinate axes
$\lvert\boldsymbol{a}\rvert$	the magnitude of $\boldsymbol{a}$
$\lvert\overrightarrow{\mathrm{AB}}\rvert$	the magnitude of $\overrightarrow{\mathrm{AB}}$
$\boldsymbol{v} \cdot \boldsymbol{w}$	the scalar product of $\boldsymbol{v}$ and $\boldsymbol{w}$
$\mathrm{P}(A)$	probability of event A
$\mathrm{P}(A')$	probability of the event "not A"
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
$x_1, x_2, \ldots$	Observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\binom{n}{r}$	number of ways of selecting r items from n items
$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
$X \sim \mathrm{B}(n, p)$	the random variable X has a binomial distribution with parameters n and p
$X \sim \mathrm{N}(\mu, \sigma^2)$	the random variable X has a normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	mean of a set of data, $x_1, x_2, x_3, \ldots$

z	standardized normal random variable, $z = \dfrac{x-\mu}{\sigma}$
Φ	cumulative distribution function of the standardized normal variable with distribution $\mathrm{N}(0,\ 1)$
r	Pearson's product–moment correlation coefficient